LMS-Bath Symposium "Operators, Asymptotics, Waves"

24 July – 4 August 2023

Department of Mathematical Sciences, University of Bath

Week 1: Workshop

- **All Week 1 talks will take place in Wolfson Lecture Theatre 4W1.7**
- **All lunches and tea/coffee breaks will take place in Level 1 Atrium, 4W**
- **Rooms 4W1.1, 4W1.2 are available for discussions and private work**

Monday 24 July

9:30–9.55 Arrival, tea/coffee: **Level 1 Atrium, 4W**

9:55–10:00 Welcome **Wolfson Lecture Theatre 4W1.7**

10:00–11:15 (60-minute talk) Stephen Shipman (Louisiana State University): Recent studies on the Fermi surface for periodic operators

11:15–11:45 Tea/coffee

11:45–12:30 (30-minute talk) Matteo Capoferri (Heriot-Watt): Curl and asymmetric pseudodifferential projections

12:30–13:45 Lunch

13:45–14:30 (30-minute talk) Sabine Bögli (Durham University): Numerical ranges and multiplier tricks

14:30–15:15 (30-minute talk) Tristan Lawrie (University of Nottingham): A quantum-graph approach to metamaterial design

15:15–15:45 Tea/coffee

15:45–17:00 (60-minute talk) Julius Kaplunov (Keele University): Asymptotic analysis of 3D dynamic problems for thin elastic plates

Tuesday 25 July

Wolfson Lecture Theatre 4W1.7

10:00–11:15 (60-minute talk) Lucas Chesnel (INRIA Paris): Maxwell's equations with hypersingularities at a negative index material conical tip

11:15–11:45 Tea/coffee

11:45–12:30 (30-minute talk) Marta de León-Contreras (Universidad de la Laguna): Variation and oscillation operators associated with discrete Jacobi operators

12:30–13:45 Lunch

13:45–14:30 (30-minute talk) Danila Prikazchikov (Keele University): Asymptotic formulations for surface and interfacial elastic waves using pseudo-differential operators

14:30–15:15 (30-minute talk) Mahran Rihani (École Polytechnique): Homogenization of Maxwell's equations and related scalar problems with sign-changing coefficients

15:15–15:45 Tea/coffee

15:45–17:00 (60-minute talk) Valery Smyshlyaev (University College London): Two-scale micro-resonant homogenisation: examples, convergence analysis and generalisations I

Wednesday 26 July

Wolfson Lecture Theatre 4W1.7

10:00–11:15 (60-minute talk) Alexander Figotin (University of California Irvine): Spectral Theory of Coupled-cavity traveling wave tubes and multicavity klystrons

11:15–11:45 Tea/coffee

11:45–12:30 (30-minute talk) Francesco Ferraresso (University of Sassari): Spectral analysis of dissipative Maxwell systems

12:30–13:45 Lunch

13:45–18:30 Free time

18:30 for 19:00 Dinner at Woods Restaurant

Thursday 27 July

Wolfson Lecture Theatre 4W1.7

10:00–11:15 (60-minute talk) Hyeonbae Kang (Inha University): A decomposition theorem of surface vector fields and spectral structure of the Neumann-Poincaré operator in elasticity

11:15–11:45 Tea/coffee

11:45–12:30 (30-minute talk) Elena Cherkaev (University of Utah): Padé approximations of Herglotz functions in homogenization and composite materials

12:30–13:45 Lunch

13:45–14:30 (30-minute talk) Mikhail Cherdantsev (Cardiff University): High-contrast random composites: homogenisation framework and new spectral phenomena

14:30–15:15 (30-minute talk) Ludmila Prikazchikova (Keele University): A generalization of Saint-Venant's principle for high-contrast elastic laminates

15:15–15:45 Tea/coffee

15:45–17:00 (60-minute talk) Ilia Kamotski (University College London): Two-scale micro-resonant homogenisation: examples, convergence analysis and generalisations II

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Friday 28 July

Wolfson Lecture Theatre 4W1.7

10:15–11:00 (30-minute talk) Josip Žubrinić (University of Zagreb): Norm-resolvent asymptotics for the system of linearized elasticity in the setting of high-contrast homogenisation

11:00–11:30 Tea/coffee

11:30–12:15 (30-minute talk) Ruming Zhang (TU Berlin): Spectrum decomposition of translation operators in periodic waveguides

12:30–13:45 Lunch

13:45–14:30 (30-minute talk) Jean Lagacé (King's College London): Homogenisation as a control — Spectral shape optimisation via homogenisation on manifolds

14:15–14:45 Tea/coffee

14:45–15:30 (30-minute talk) Shota Fukushima (Inha University): Spectrum of Neumann-Poincaré operators on the boundary with axial symmetry

15:30–16:00 Tea/coffee

End of Week 1

Week 1: Abstracts of Talks

Sabine Bögli (Durham University): Numerical ranges and multiplier tricks

Instead of solving the eigenvalue problem $Tf = zf$ for a linear operator T and eigenvalue z, we can use a multiplier B and instead solve the linear pencil problem $BTf = zBf$. This leads us to study the numerical range and essential numerical range of linear pencils. The essential numerical range is used to describe the set of spectral pollution when approximating the eigenvalue problem by projection and truncation methods. By taking intersection over various multipliers, we get sharp enclosures. We apply the results to various differential operators. This talk is based on joint work with Marco Marletta.

Matteo Capoferri (Heriot-Watt): Curl and asymmetric pseudodifferential projections

In my talk I will present a new approach to the spectral theory of systems of PDEs on closed manifolds, developed in a series of recent papers by Dmitri Vassiliev (UCL) and myself, based on the use of pseudodifferential projections. After discussing the general theory, I will turn to the (non-elliptic) operator curl, and explain how our techniques offer a new pathway to the study of spectral asymmetry.

Mikhail Cherdantsev (Cardiff University): High-contrast random composites: homogenisation framework and new spectral phenomena

We study the homogenisation problem for elliptic operators in the divergence form with high-contrast random coefficients. In particular, we are interested in the behaviour of their spectra. We assume that on one of the components of the composite the coefficients are "of order one", the complimentary "soft" component consists of randomly distributed inclusions, whose size and spacing are small, and the coefficients on the soft component exhibit the so called double-porosity scaling.

Our interest in high-contrast homogenisation problems is motivated by the band-gap structure of their spectra. From an intuitive point of view, this phenomenon can be explained by viewing the "soft" inclusions as micro-resonators, which may dramatically amplify of completely block the propagation of waves in the medium, depending on the frequency. From a mathematically rigorous perspective, this was first analysed by Zhikov (2000, 2004) in the periodic setting.

In this talk I will present our recent results and the ongoing work in this area, including the scalar case, linear elasticity systems and eigenfunction localisation on a defect.

Elena Cherkaev (University of Utah): Padé approximations of Herglotz functions in homogenization and composite materials

Padé approximations of Stieltjes and Herglotz-Nevanlinna functions play a central role in the operator theory connecting the spectral theory with the moment problem and continuous fractions, Jacobi matrices, orthogonal polynomials, and approximation problems. The talk will discuss matrix Padé approximations of the resolvents of operators arising in a homogenization problem. Stieltjes/Herglotz function integral representation of the homogenized transport coefficients links the microgeometry of a composite material to the spectral measure of a related self-adjoint operator. I will show that a matrix spectral measure in the integral representation of the effective properties of the composite can be uniquely reconstructed; this uniqueness provides a basis for the inverse homogenization problem of recovering information about the microgeometry of the medium. Padé approximations of the spectral measure allow constructing bounds for the effective properties of composites in forward homogenization and result in spectrally matched geometries in inverse homogenization, linking it to the inverse spectral problem. In addition, a Padé approximant provides a set of internal or hidden variables naturally related to the material's structure. These hidden variables determine the characteristic internal scales corresponding to the microlevel processes responsible for the dispersion and dissipation phenomena in wave propagation through a microstructured medium.

Lucas Chesnel (INRIA Paris): Maxwell's equations with hypersingularities at a negative index material conical tip

In this talk, we are interested in the analysis of time-harmonic Maxwell's equations in presence of a conical tip of a material with negative dielectric constants. When these constants belong to some critical range, the electromagnetic field exhibits strongly oscillating singularities at the tip which have infinite energy. Consequently Maxwell's equations are not well-posed in the classical L^2 framework. The goal of the present work is to provide an appropriate functional setting for 3D Maxwell's equations when both the dielectric permittivity and the magnetic permeability take critical values. Following what has been done for the 2D scalar case, the idea is to work in weighted Sobolev spaces, adding to the space the so-called outgoing propagating singularities. The analysis requires new results of scalar and vector potential representations of singular fields. The outgoing behaviour is selected via the limiting absorption principle.

Francesco Ferraresso (University of Sassari): Spectral analysis of dissipative Maxwell systems

Electromagnetic waves propagating through conductive media tend to lose part of their energy; from a mathematical point of view, conductivity makes the underlying Maxwell operator non-selfadjoint. I will discuss some recent results about the spectrum of the anisotropic dissipative Maxwell system in (possibly) unbounded domains of $\mathbb{R}^3$.
Assuming that the tensors ε, μ tend to real multiples of the identity matrix at infinity, the essential spectrum is characterised as the union of two parts, one related to the behaviour of divergence-free vector fields 'at infinity'; the other to the loss of ellipticity of a suitably defined $\operatorname{div} p(\omega)\nabla$ operator. Spectral approximation via domain truncation is shown to be reliable outside an explicit, 'small' set of spectral pollution. Time-permitting, I will discuss some generalisations to non-constant coefficients 'at infinity'.
Based on joint work with S. Bögli, M. Marletta, and C. Tretter.

Alexander Figotin (University of California Irvine): Spectral Theory of Coupled-cavity traveling wave tubes and multicavity klystrons

Coupled-cavity traveling wave tube (CCTWT) is a high power microwave vacuum electronic device used to amplify radio-frequency (RF) signals. CCTWTS have numerous applications, including radar, radio navigation, space communication, television, radio repeaters, and charged particle accelerators. The microwave-generating interactions in CCTWTs take place mostly in coupled resonant cavities positioned periodically along the electron beam axis. Operational features of a CCTWT particularly the amplification mechanism are similar to those of a multicavity klystron (MCK). We discuss in this presentation a Lagrangian field theory of CCTWTs with the space being represented by one-dimensional continuum. The theory integrates into it the space-charge effects including the so-called debunching (electron-to-electron repulsion). The corresponding Euler-Lagrange field equations are ODEs with coefficients varying periodically in the space. Utilizing the system periodicity we develop the instrumental features of the Floquet theory including the monodromy matrix and its Floquet multipliers. We use them to derive closed form expressions for a number of physically significant quantities. Those include in particular the dispersion relations and the frequency dependent gain foundational to the RF signal amplification. Serpentine (folded, corrugated) traveling wave tubes are very similar to CCTWTs and our theory applies to them also.

Shota Fukushima (Inha University): Spectrum of Neumann-Poincaré operators on the boundary with axial symmetry

We consider the spectra of Neumann-Poincaré operators on boundaries of three-dimensional domains with axial symmetries like tori. The Neumann-Poincaré operator on the axially symmetric domain is decomposed by the Fourier expansion. Then each component of the Neumann-Poincaré operator is approximated by the Neumann-Poincaré operator and single layer potential on the two-dimensional cross section. In particular, the essential spectrum of the axially symmetric domain includes that of the Neumann-Poincaré operator on the two-dimensional cross section. This talk is based on the joint work with Hyeonbae Kang (Inha University).

Ilia Kamotski (University College London): Two-scale micro-resonant homogenisation: examples, convergence analysis and generalisations II

A motivating now classical example is wave propagation in periodic media with 'soft' inclusions surrounded by 'stiff' matrices, where the inclusions serve as subwavelength micro-resonators. This corresponds to PDE models with a special critical scaling between the small periodicity and the high contrast, and the resulting two scale approximations for the solutions remain intrinsically two-scale reflecting the underlying resonance effect. Mathematically this leads to studying asymptotic properties of Floquet-Bloch spectral problems and of the underlying operators. We review the background as well as some more recent advances related to two-scale operator and spectral convergence, error bounds, and further generalisation based on more abstract general theories illustrated by diverse examples. Parts of this longer-term project are done in joint collaborations between Ilia Kamotski, Valery Smyshlyaev and Shane Cooper.

Hyeonbae Kang (Inha University): A decomposition theorem of surface vector fields and spectral structure of the Neumann-Poincare operator in elasticity

We prove that the space of vector fields on the boundary of a bounded domain with the Lipschitz boundary in three dimensions is decomposed into three subspaces: elements of the first one extend to the inside the domain as divergence-free and rotation-free vector fields, the second one to the outside as divergence-free and rotation-free vector fields, and the third one to both the inside and the outside as divergence-free harmonic vector fields. We apply this decomposition theorem to investigate spectral properties of the Neumann-Poincaré operator in elasticity, whose cubic polynomial is known to be compact. We prove that each linear factor of the cubic polynomial is compact on each subspace of decomposition separately and those subspaces characterize eigenspaces of the Neumann-Poincaré operator. This is a joint work with S. Fukushima and Y.-G. Ji.

Julius Kaplunov (Keele University): Asymptotic analysis of 3D dynamic problems for thin elastic plates

Modern challenges typical for the asymptotic derivation of low-dimensional dynamic models for thin elastic plates are considered. In particular, pseudo-differential boundary conditions along plate faces modelling fluid-structure interaction are studied. Higher order approximations of the original equations of motion are constructed. Physically motivated error estimates are presented. The relevance of a multiparametric analysis is demonstrated. Other challenges of the general asymptotic theory, including the formulation of consistent boundary and initial conditions, as well as analysis of high-frequency wave propagation, are addressed.

Jean Lagacé (King's College London): Homogenisation as a control — Spectral shape optimisation via homogenisation on manifolds

Traditionally, homogenisation theory uses some form of translation invariance in order to describe some PDE with fast oscillations, making it unsuitable for a general study of phenomena on manifolds. In this talk, I will describe one way to adapt the theory to the Riemannian setting, and how we could use the homogenisation process as a control in order to reach the boundary of parameter space in a spectral shape optimisation problem, namely for the Laplace and Steklov problems. In particular, this highlights some relationships in the theory of minimal surfaces with and without boundary.
This is joint work with Antoine Henrot (Nancy), Alexandre Girouard (Laval) and Mikhail Karpukhin (UCL)

Tristan Lawrie (University of Nottingham): A quantum-graph approach to metamaterial design

Since the turn of the century, metamaterials have gained a large amount of attention due to their potential for possessing highly nontrivial and exotic properties?such as cloaking or perfect lensing. There has been a great push to create reliable mathematical models that accurately describe the required material composition. Here, we consider a quantum graph approach to metamaterial design. An infinite square periodic quantum graph, constructed from vertices and edges, acts as a paradigm for a 2D metamaterial. Wave transport occurs along the edges with vertices acting as scatterers modelling sub-wavelength resonant elements. These resonant elements are constructed with the help of finite quantum graphs attached to each vertex of the lattice with customisable properties controlled by a unitary scattering matrix. The metamaterial properties are understood and engineered by manipulating the band diagram of the periodic structure. The engineered properties are then demonstrated in terms of the reflection and transmission behaviour of Gaussian beam solutions at an interface between two different metamaterials. We extend this treatment to N layered metamaterials using the Transfer Matrix Method. We demonstrate both positive and negative refraction and beam steering. Our proposed quantum graph modelling technique is very flexible and can be easily adjusted making it an ideal design tool for creating metamaterials with exotic band diagram properties or testing promising multi-layer set ups and wave steering effects.

Marta de León-Contreras (Universidad de la Laguna): Variation and oscillation operators associated with discrete Jacobi operators

Let $\alpha, \beta \geq -1/2$ and consider the discrete Jacobi operator

$$J^{(\alpha,\beta)}(f)(n) = a_{n-1}^{(\alpha,\beta)} f(n-1) + b_n^{(\alpha,\beta)} f(n) + a_n^{(\alpha,\beta)} f(n+1), \quad n \in \mathbb{N},$$

where $a_n^{(\alpha,\beta)}, b_n^{(\alpha,\beta)}$ are certain coefficients. This operator is a generalization of the ultraspherical operator (and therefore of the discrete Laplacian). In this talk we will see how to prove weighted ℓ^p-inequalities for variation and oscillation operators defined by the discrete Jacobi heat semigroup, $\{e^{tJ^{(\alpha,\beta)}}\}_{t>0}$. These ℓ^p-boundedness properties provide information about the *a.e.* convergence of the semigroup as $t \to 0$.

Danila Prikazchikov (Keele University): Asymptotic formulations for surface and interfacial elastic waves using pseudo-differential operators

The contribution is focused on a new interpretation of the hyperbolic-elliptic models for Rayleigh, Schölte-Gogoladse and Stoneley waves excited by prescribed tractions at the boundary, resulting in a hyperbolic PDE at a given depth, with the forcing involving a pseudo-differential operator containing the normal coordinate as a parameter. An example of an impulse concentrated load is considered, revealing the smoothening delta-like behaviour.

Ludmila Prikazchikova (Keele University): A generalization of Saint-Venant's principle for high-contrast elastic laminates

The Saint-Venant's principle is a basic concept of mathematical elasticity. It manifests an exponential decay of the stress fields, corresponding to self-equilibrated edge loads. There are very few examples of the exact solutions validating the Saint-Venant's principle, including the plane strain problem for an elastic semi-infinite strip. An asymptotically perturbed formulation of the canonical Saint-Venant's principle is central for deriving consistent boundary conditions in the general theory for thin elastic shells. A more recent asymptotic extension of this principle is oriented to low-frequency dynamics of an elastic semi-infinite strip. In this presentation we further generalize the Saint-Venant's principle, adapting it for high-contrast laminates. In particular, the antiplane shear of a semi-infinite multi-layered elastic strip, composed of alternating stiff and soft layers, is studied. An asymptotic approach is developed to derive explicit conditions on the edge load, ensuring the decay of stress components at the distance of the order of strip thickness. One of these conditions agrees with the Saint-Venant's principle for a homogeneous elastic solid. The rest of them are due to presence of a high contrast. Their number is equal to that of soft layers. As an illustration, the obtained decay conditions are implemented for calculating the interior solution for a three-layered sandwich.

Mahran Rihani (École Polytechnique): Homogenization of Maxwell's equations and related scalar problems with sign-changing coefficients

In this work, we are interested in the homogenization of time-harmonic Maxwell's equations in a composite medium with periodically distributed small inclusions of a negative material. Here a negative material is a material modelled by negative permittivity and permeability. Due to the sign-changing coefficients in the equations, it is not straightforward to obtain uniform energy estimates to apply the usual homogenization techniques. The goal of this article is to explain how to proceed in this context. The analysis of Maxwell's equations is based on a precise study of two associated scalar problems: one involving the sign-changing permittivity with Dirichlet boundary conditions, another involving the sign-changing permeability with Neumann boundary conditions. For both problems, we obtain a criterion on the physical parameters ensuring uniform invertibility of the corresponding operators as the size of the inclusions tends to zero. In the process, we explain the link existing with the so-called Neumann-Poincaré operator complementing the existing literature on this topic. Then we use the results obtained for the scalar problems to derive uniform energy estimates for Maxwell's system. At this stage, an additional difficulty comes from the fact that Maxwell's equations are also sign-indefinite due to the term involving the frequency. To cope with it, we establish some sort of uniform compactness result.

Stephen Shipman (Louisiana State University): Recent studies on the Fermi surface for periodic operators

The Bloch and Fermi varieties are relations between energy and momentum (or frequency and wavevector). The Bloch variety is essentially the dispersion relation for a periodic operator. I will present recent work and interesting problems concerning these objects, highlighting their structure as algebraic or analytic varieties. I will give a rough survey of results on band degeneracy, inverse problems, and reducibility, with a focus on the latter and its consequences for defect states in the continuum.

Valery Smyshlyaev (University College London): Two-scale micro-resonant homogenisation: examples, convergence analysis and generalisations I

A motivating now classical example is wave propagation in periodic media with 'soft' inclusions surrounded by 'stiff' matrices, where the inclusions serve as subwavelength micro-resonators. This corresponds to PDE models with a special critical scaling between the small periodicity and the high contrast, and the resulting two scale approximations for the solutions remain intrinsically two-scale reflecting the underlying resonance effect. Mathematically this leads to studying asymptotic properties of Floquet-Bloch spectral problems and of the underlying operators. We review the background as well as some more recent advances related to two-scale operator and spectral convergence, error bounds, and further generalisation based on more abstract general theories illustrated by diverse examples. Parts of this longer-term project are done in joint collaborations between Ilia Kamotski, Valery Smyshlyaev and Shane Cooper.

Ruming Zhang (TU Berlin): Spectrum decomposition of translation operators in periodic waveguides

Scattering problems in periodic waveguides are interesting but also challenging topics in mathematics both theoretically and numerically. As is well known, the unique solvability of these problems is not always guaranteed. To obtain a unique solution that is "physically meaningful," the limiting absorption principle (LAP) is a commonly used method. LAP assumes that the limit of a family of solutions with absorbing media converges as the absorption parameter tends to zero and that the limit is the "physically meaningful solution." It is also called the LAP solution in this talk. It has been proved that the LAP holds for periodic waveguides in [V. Hoang, SIAM J. Appl. Math., 71 (2011), pp. 791–810]. In this talk, we consider the spectrum decomposition of periodic translation operators. With the curve integral formulation and a generalized residue theorem, the operator is explicitly described by its eigenvalues and generalized eigenfunctions, which are closely related to Bloch wave solutions. Then the LAP solution is decomposed into generalized eigenfunctions. This gives a better understanding of the structure of scattered fields.

Josip Žubrinić (University of Zagreb): Norm-resolvent asymptotics for the system of linearized elasticity in the setting of high-contrast homogenisation

We provide norm-resolvent asymptotics for the system of partial differential equations describing the periodically heterogeneous elastic medium with highly oscillating coefficients in high contrast.
The analysis is carried out by employing the technique of boundary triples and spectral analysis of the underlying Dirichlet-to-Neumann operators. We obtain explicit dispersion relations which display the meta-material properties of the effective medium, such as the time non-locality and spectral gaps.
This is joint work with K. Cherednichenko, A. Kiselev and I. Velčić.

Week 2: Research Incubator

- **All group activities will take place in 4W1.7, 4W1.2**
- **All lunches will take place in Level 1 Atrium, 4W. Tea/coffee are available throughout the day.**
- **Room 4W1.1 is available for discussions and private work**

Monday 31 July

Morning session 9:30–12:30

Julius Kaplunov: Asymptotic near-cut-off analysis of a thin elastic plate resting on a Winkler foundation

Hyeonbae Kang: Singularly continuous spectrum

9:30–9:45	Joint presentation of 2 topics
9:45–10:00	Split presentations: 2 topics in detail
10:00–11:00	Group work
11:00–11:30	Break
11:30–12:30	Group work

12:30–14:00 Lunch

Afternoon session 14:00–17:00

Stephen Shipman: Complex Bragg resonance for periodic graph operators

Valery Smyshlyaev: Whispering gallery wave scattering by boundary inflection: an unsolved canonical problem in asymptotics

14:00–14:15	Joint presentation of 2 topics
14:15–14:30	Split presentations: 2 topics in detail
14:30–15:30	Group work
15:30–16:00	Break
16:00–17:00	Group work

Tuesday 1 August

Morning session 9:30–12:30

Alexander Figotin: Expanding the conventional spectral theory by the calculus of variations

Ilia Kamotski: Approximations in classical homogenisation theory

9:30–9:45	Joint presentation of 2 topics
9:45–10:00	Split presentations: 2 topics in detail
10:00–11:00	Group work
11:00–11:30	Break
11:30–12:30	Group work

12:30–14:00 Lunch

Afternoon session 14:00–17:00

Breakaway group work

Wednesday 2 August

Morning session 9:30–12:30

VALERY SMYSHLYAEV: Defect resonances in high-contrast periodic media and localisation-like phenomena

JULIUS KAPLUNOV: Low–frequency vibrations of a thin functionally graded plate

9:30–9:45	JOINT PRESENTATION OF 2 TOPICS
9:45–10:00	SPLIT PRESENTATIONS: 2 TOPICS IN DETAIL
10:00–11:00	GROUP WORK
11:00–11:30	BREAK
11:30–12:30	GROUP WORK

12:30–14:00 LUNCH

Afternoon: FREE TIME

Thursday 3 August

Morning session 9:30–12:30

Ilia Kamotski: Some old and new problems in high contrast homogenisation

Stephen Shipman: Asymmetric bound states in the continuum for PDEs

9:30–9:45	Joint presentation of 2 topics
9:45–10:00	Split presentations: 2 topics in detail
10:00–11:00	Group work
11:00–11:30	Break
11:30–12:30	Group work

12:30–14:00 Lunch

Afternoon session 14:00–17:00

Hyeonbae Kang: Spectral property of the Neumann-Poincaré operator for cloaking by anomalous localized resonance

Alexander Figotin: Exactly solvable models for complex continuum systems

14:00–14:15	Joint presentation of 2 topics
14:15–14:30	Split presentations: 2 topics in detail
14:30–15:30	Group work
15:30–16:00	Break
16:00–17:00	Group work

Friday 4 August

Morning session 9:30–12:30

Summary day: Part 1

12:30–14:00 Lunch

Afternoon session 14:00–17:00

Summary Day: Part 2

End of Week 2

Week 2: Outlines of Discussion Sessions

Alexander Figotin

Tuesday morning: Expanding the conventional spectral theory by the calculus of variations

The classical spectral is theory is devised to analyze physical systems evolution governed by relevant self-adjoint operators. This theory is a linear one and it covers mostly the systems describing small oscillations about a stable equilibrium. This situation occurs when the system evolves in a small vicinity of a stable equilibrium and the forces always act to restore the equilibrium whenever the system deviates from it. There are important physical systems including the various plasmas that involve instabilities and significant nonlinear phenomena. These systems are not covered by the conventional spectral theory. In this discussion we would like to consider possibilities to expand the conventional spectral theory to account for an unstable evolution that may involve waves growing exponentially either in time or in space. We suggest to consider a possible expansion of the conventional spectral theory based on the physically sound Hamilton principle of least action which is a part of the Calculus of Variations. This approach allows also to consider nonlinear evolution featuring non-quadratic Lagrangians. We demonstrate the power and the efficiency of the proposed approach applying it to some systems involving non-neutral plasma such as electron beams in traveling tubes and klystrons.

Thursday afternoon: Exactly solvable models for complex continuum systems

In this discussion we suggest some non-trivial exactly solvable models of continua based on advanced by us constructive Floquet-Bloch theory. These models describe the evolution of multicavity klystrons (MCKs) and coupled-cavity traveling wave tubes (CCTWTs). They are constructed based on the Hamilton principle of least action yielding relevant Lagrangian field theories. The corresponding Euler-Lagrange field equations are ODEs with coefficients varying periodically in the space. Utilizing the system periodicity we develop the instrumental features of the Floquet theory including the monodromy matrix and relevant Floquet multipliers. We use them to derive closed form expressions for a number of physically significant quantities. including in particular the dispersion relations. Using the above models as examples we would like to discuss possibilities for extending the spectral theory of periodic system to account for new phenomena such as various instabilities and as well non-trivial dissipation effects including the dissipation caused instability.

Ilia Kamotski

Tuesday morning: Approximations in classical homogenisation theory

It is well known that solutions to homogenisation problems for scalar second order differential operators may be approximated in the spaces H^1 and L^2 with appropriate rates. There is flexibility in how we choose these approximations. Some may be better than others depending on what we want from the approximation. I am going to discuss one particular (probably new) approximation. Higher order approximations remain a challenge from this perspective.

Thursday morning: Some old and new problems in high contrast homogenisation

I am planning to continue discussion of general scheme for high contrast homogenisation problems presented last week and which is based on joint work with S. Cooper and V. Smyshlyaev. I will demonstrate what the proposed general scheme gives in specific examples, point out possible variants of its improvements and generalisations, as well as it's shortcomings. Possible further developments will be discussed.

Hyeonbae Kang

Monday morning: Singularly continuous spectrum

The essential spectrum of the Neumann-Poincaré operator on two-dimensional curvilinear polygons consists of absolutely continuous spectrum and singularly continuous spectrum is void (Perfekt-Putinar, Perfekt, Kang-Lim-Yu). Singularly continuous spectrum remains mysterious, as no geometric condition is known for existence of singularly continuous spectrum for the Neumann-Poincaré operator.

This symposium will provide a good opportunity to discuss singularly continuous spectrum from various areas of research, which may include Schrödinger operator (Simon, Last, . . .) and Weyl-von Neumann theorem for instability of absolutely continuous spectrum.

Thursday afternoon: Spectral property of the Neumann-Poincaré operator for cloaking by anomalous localized resonance

Cloaking by anomalous localized resonance (CALR) is a resonance phenomenon occurring at the limit point of eigenvalues of the Neumann-Poincaré operator, and can be characterized quantitatively using its spectral information. It is known in two dimensions that CALR occurs on annuli and ellipses. It is known in three dimensions that CALR does not occur on strictly convex domains. No class of three-dimensional domains where the CALR takes place is known. I will briefly explain this.

Julius Kaplunov

Monday morning: Asymptotic near-cut-off analysis of a thin elastic plate resting on a Winkler foundation

The mathematical background of a classical engineering model in structural mechanics is studied. The essential problem parameters are defined. The failure of the 2D ad hoc formulation near the cut-off frequency is established. A 3D asymptotic scheme for an elastic layer with mixed boundary conditions along its faces is developed. The results for one and two-sided contacts are compared.

Wednesday morning: Low–frequency vibrations of a thin functionally graded plate

The 2D low-frequency model for a homogeneous elastic plate was asymptotically justified a long time ago. The extension to a transversely inhomogeneous, e.g. functionally graded plate faces various challenges due to asymmetry of the problem parameters with respect to the mid-plane. In particular, the separation of bending and extension deformations is of primary interest. To this end, a closer look at the peculiarities of the underlying asymptotic procedure is expected.

Stephen Shipman

Monday afternoon: Complex Bragg resonance for periodic graph operators

The Fermi variety of a periodic operator is a cross-section in quasi-momentum space (k) of the dispersion relation at a fixed energy E, that is $\{k|D(k,E)=0\}$. For a graph operator, it is an algebraic variety in the Floquet multipliers $(z_1,...,z_d)=\exp(\mathrm{i}k_1,...,\mathrm{i}k_d)$. When the Fermi variety is reducible, one can construct a local defect that supports a bound state (without compact support) at an energy embedded in the continuum. But in two or more dimensions $(d>1)$ a polynomial in z is generically not factorable, so that the Fermi variety is generically not reducible. This means that a generic periodic perturbation of a periodic operator that admits a defect state in the continuum will destroy the defect state, turning it into a resonance. Furthermore, no perturbation of the defect will be able to restore it to a true bound state. We would like to analyze the details of this resonance, including the time dynamics. The leading contribution should come from the singular set of the Fermi variety, particularly where the components intersect, and this could happen in the complex energy-momentum domain. This is a complex version of Bragg resonance, which occurs when crossing branches of the real dispersion relation merge to form a spectral gap.

Thursday morning: Asymmetric bound states in the continuum for PDEs

The easy way to create a bound state in the continuum (BIC) is to use a finite symmetry group. BIC in the absence of symmetry are associated with richer resonance phenomena, but they are hard to construct. This has been achieved for graph models and coupled-mode models, and it has been observed numerically and experimentally in optical structures. The aim of this problem is to develop a theory for BIC in the absence of symmetry for PDEs, particularly for the Maxwell system.

Valery Smyshlyaev

Monday afternoon: Whispering gallery wave scattering by boundary inflection: an unsolved canonical problem in asymptotics

When a whispering gallery wave, which is a high-frequency asymptotic mode propagating along a concave part of a boundary, approaches a boundary inflection point it breaks-up and scatters. The problem leads to an arguably as fundamental canonical boundary-value problem for a special PDE describing transition from a "modal" to a "scattered" asymptotic behaviour, as Airy functions are for transition from oscillatory to exponentially decaying asymptotic patterns. The problem was formulated by M. M. Popov in 1970-s and remains largely open since despite considerable efforts and some progress. The associated solutions have asymptotic behaviours with a discrete spectrum at one end and with a continuous spectrum at the other end. Of central interest is to find the map connecting the above two asymptotic regimes. We recently reviewed this problem and uncovered some asymptotic properties at the continuous end. We also gave some interpretations in terms of the existence of the associated generalized wave operators, and of a version of a unitary scattering operator connecting the modal and scattered asymptotic regimes. It remains unclear whether and how one can describe these more explicitly, and ultimately evaluate numerically.

Wednesday morning: Defect resonances in high-contrast periodic media and localisation-like phenomena

As we now know, perfectly periodic high-contrast "soft" inclusions in an infinite "stiff" matrix produce band gap opening near the inclusions' resonances (i.e. their Dirichlet eigenvalues, identical for every inclusion). If one allows for the inclusions' eigenvalues to vary, say by keeping their geometry perfectly periodic but introducing some random pre-factors in the inclusions' stiffness, then formal two-scale asymptotics in the time domain appears to display some interesting-looking phenomena like apparent dissipation or possibly even localisation. An open problem is what exactly this means, and whether and how one can try and analyse this rigorously. For example, we can start with altering a single inclusion, possibly also rescaling its size, so that its Dirichlet eigenvalue is in a gap of the surrounding periodic medium. Will then, for high enough contrasts, a "true" eigenvalue exist in the gap near the above Dirichlet one? If so, can one then start multiplying such inclusions filling the gaps with eigenvalues and ultimately passing to some kind of a limit?

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