# Large N via stochastic quantization

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In some Euclidean QFT models, which are defined by formal measures $e^{-\mathcal{S}(\Phi)}D\Phi$, the fields $\Phi$ take values in $N$-dim space.

In this talk, we consider stochastic quantization of such models and $N \to \infty$ questions.

We can exploit techniques from PDE, stochastic analysis, and mean field theory.

$\Phi^4$ quantum field theory

$$\exp\Big(-\int_{\mathbf{T}^d}(\nabla\Phi)^2 + m\Phi^2 + \frac{\lambda}{4}\Phi^4\Big)\mathcal{D}\Phi$$

Stochastic quantization

$$\partial_t\Phi = (\Delta - m)\Phi - \lambda\Phi^3 + \xi$$

Vector $\Phi^4$ (linear sigma model), $\Phi = (\Phi_1, \cdots, \Phi_N)$

$$\exp\Big( - \int_{\mathbf{T}^d} \sum_{i=1}^{N} (\nabla \Phi_i)^2 + m \sum_{i=1}^{N} \Phi_i^2 + \frac{\lambda}{N} (\sum_{i=1}^{N} \Phi_i^2)^2 \Big) \mathcal{D}\Phi$$

Stochastic quantization

$$\partial_t \Phi_i = (\Delta - m)\Phi_i - \frac{\lambda}{N} \sum_{j=1}^{N} \Phi_j^2 \Phi_i + \xi_i \qquad (i = 1, \cdots, N)$$

**Question:** Asymptotic behavior as $N \to \infty$. (Mean Field Limit)

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## Physics background.

- [Stanley'68] Spin models, converge to solvable model as $N \to \infty$
- [Wilson'73] [Gross-Neveu'74] $N$-component $\Phi^4$ and fermionic QFTs
- [t'Hooft'74] $SU(N)$ Yang-Mills, dual to strings
- Migdal, Witten, ... since '79
- nonlinear $\sigma$-models, target space $N$ dimensional..................

## Some math results.

[Levy'11][Anshelevich-Sengupta'12] YM in 2d (some "integrability")

[Chatterjee-Jafarov'16] lattice YM (continuum would be harder than lattice).

[Kupiainen'80] certain observables in $N$-component $\Phi^4$ in 2d

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**Theorem 1 (Mean field limit in non-equilibrium).** $d = 2$.

$$\partial_t \Phi_i = (\Delta - m)\Phi_i - \frac{\lambda}{N}\sum_{j=1}^{N} :\Phi_j^2 \Phi_i: \ + \xi_i$$

$$\partial_t \Psi_i = (\Delta - m)\Psi_i - \lambda :\mathbf{E}(\Psi_i^2): \ \Psi_i + \xi_i$$

$$\lim_{N\to\infty} \mathbf{E}\|\Phi_i - \Psi_i\|_{L^2}^2 = 0$$

**Rmk:** Vast math literature on mean field limits for SDE systems, or interacting particles; but not on singular SPDEs. For example:

$$\text{(Sznitman 80s)} \qquad dx_i = \frac{1}{N}\sum_{j=1}^{N} f(x_i, x_j)dt + d\beta_i$$

$$\overset{N\to\infty}{\to} \qquad dy_i = \int f(y_i, z)\mu_t(dz) + d\beta_i \qquad \mu_t = \mathrm{law}(y_i(t))$$

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Theorem 2 (QFT large N limit). $d = 2, 3$. For $m$ large or $\lambda$ small,

$$\exp\Big( - \int_{\mathbf{T}^d} \sum_{i=1}^N (\nabla \Phi_i)^2 + m \sum_{i=1}^N \Phi_i^2 + \frac{\lambda}{N} :(\sum_{i=1}^N \Phi_i^2)^2: \Big) \mathcal{D}\Phi$$

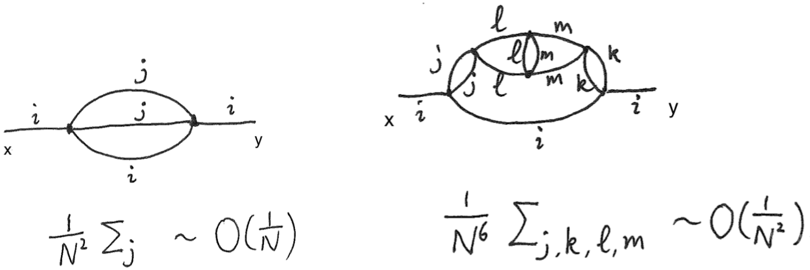
$$\xrightarrow{N\to\infty} \qquad \mathrm{GFF}((m-\Delta)^{-1}) \quad \sim \quad e^{-\int_{\mathbf{T}^d} (\nabla\Psi)^2 + m\Psi^2} \mathcal{D}\Psi$$

**Rmk.** Predicted by perturbation theory e.g. Wilson'73

(Proof uses dynamics)

Heuristics of [Wilson'73]:

- Interaction is $\frac{1}{N}\sum_{j,k=1}^{N} {:}\Phi_j^2\Phi_k^2{:}$
- Diagrams for $\mathbf{E}(\Phi_i(x)\Phi_i(y))$: each vertex is degree 4, with indices of the form $(j, j, k, k)$ on the four edges.




Only leading order ("zero loop") diagrams for correlations are $O(1)$, all other diagrams are $O(\frac{1}{N})$ or smaller. (There're also 'dual field' argument etc.)

Theorem 3 (Observables). $d = 2$. Let $\mathcal{O} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} :\Phi_i^2:$ Consider $G_N(x-y) = \mathbf{E}(\mathcal{O}(x)\mathcal{O}(y))$. We have

$$\widehat{G_N} \stackrel{N\to\infty}{=} \frac{2\widehat{C^2}}{1+2\widehat{C^2}} \qquad C = (m-\Delta)^{-1}$$

**Rmk.** Let $Z_i$ be i.i.d. GFF and $\mathcal{O}' = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} :Z_i^2:$ , then

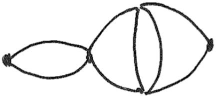
$$\mathbf{E}(\mathcal{O}'(x)\mathcal{O}'(y)) = 2C^2(x-y)$$

**Rmk.** Predicted by perturbation theory; rigorously by [Kupiainen'80]

(Proof uses Dyson-Schwinger equations + estimates on dynamics)

Perturbative calculations for $\mathbf{E}(\mathcal{O}(x)\mathcal{O}(y))$ where $\mathcal{O} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} :\Phi_i^2:$

$\sim O(1)$



$\sim O(\frac{1}{N})$

## Review for $N = 1$ case on $\mathbf{T}^2$

$$\partial_t \Phi = (\Delta - m)\Phi - \lambda :\Phi^3: + \xi \qquad \text{Decompose: } \Phi = Z + Y$$

$$\partial_t Z = (\Delta - m) Z + \xi \qquad Z \in C^{0-}$$

$$\partial_t Y = (\Delta - m) Y - \lambda (Y^3 + 3Y^2 Z + 3Y :Z^2: + :Z^3:)$$

Local theory: $Z,\ :Z^2:,\ :Z^3: \in C^{0-}$ and $Y \in C^{2-}$

Global control: [Mourrat-Weber'17]

$$\partial_t \|Y\|_{L^2}^2 + \|\nabla Y\|_{L^2}^2 + \lambda \|Y\|_{L^4}^4 = -\lambda \left\langle Y,\ 3Y^2 Z + 3Y :Z^2: + :Z^3: \right\rangle$$

Right hand side is bounded by $\frac{1}{5}\|\nabla Y\|_{L^2}^2 + \frac{\lambda}{5}\|Y\|_{L^4}^4 + C$

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## Proof of Theorem 1 (mean field limit)

- Above solution theory easily extends to fixed $N$

$$\partial_t \Phi_i = (\Delta - m)\Phi_i - \frac{\lambda}{N}\sum_{j=1}^{N} :\Phi_j^2 \Phi_i: \; + \xi_i$$

- For limiting SPDE

$$\partial_t \Psi = (\Delta - m)\Psi - \lambda :\mathbf{E}(\Psi^2): \Psi + \xi$$

$$\Psi = Z + X \qquad \partial_t Z = (\Delta - m)Z + \xi$$

Then since $:\mathbf{E}(\Psi^2): \stackrel{def}{=} \mathbf{E}\Psi^2 - \mathbf{E}Z^2 = \mathbf{E}[X^2 + 2ZX]$ one has

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## Proof of Theorem 1 (mean field limit)

$$\Phi_i = Z_i + Y_i \qquad \Psi_i = Z_i + X_i$$

$$\mathcal{L} Z_i = \xi_i \qquad \mathcal{L} = (\partial_t - \Delta + m)$$

Our goal is to show $\mathbf{E}\|Y_i - X_i\|_{L^2}^2 \stackrel{N\to\infty}{\to} 0$ where

$$\mathcal{L} Y_i = -\frac{\lambda}{N}\sum_{j=1}^{N}\Big(Y_j^2 Y_i + Y_j^2 Z_i + 2Y_j Y_i Z_j + 2Y_j :Z_i Z_j: + :Z_j^2: Y_i + :Z_i Z_j^2:\Big)$$

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Lemma (Uniform in $N$ estimate)

$$\frac{1}{N}\mathbf{E}\sup_{t\in[0,T]}\sum_{j=1}^{N}\|Y_j\|_{L^2}^2+\frac{1}{N}\mathbf{E}\sum_{j=1}^{N}\|\nabla Y_j\|_{L_T^2L^2}^2+\mathbf{E}\left\|\frac{1}{N}\sum_{i=1}^{N}Y_i^2\right\|_{L_T^2L^2}^2\leq C$$

Lemma (Estimate for the limit)

$$\sup_{t\in[0,T]}\mathbf{E}\|X_i\|_{L^p}^p+\mathbf{E}\||X_i|^{\frac{p-2}{2}}\nabla X_i\|_{L_T^2L^2}^2+\|\mathbf{E}|X_i|^p\mathbf{E}X_i^2\|_{L_T^1L^1}\leq C$$

Lemma (Estimate difference) Let $v_i=Y_i-X_i$

$$\frac{d}{dt}\sum_{i=1}^{N}\|v_i\|_{L^2}^2+\sum_{i=1}^{N}\|\nabla v_i\|_{L^2}^2+m\sum_{i=1}^{N}\|v_i\|_{L^2}^2+\frac{1}{N}\sum_{i,j=1}^{N}\|Y_jv_i\|_{L^2}^2+\frac{1}{N}\left\|\sum_{j=1}^{N}X_jv_j\right\|_{L^2}^2=\text{ten terms}$$

**Rmk.** Mean-zero independent $U_1,\dots,U_N$, $\mathbf{E}\|\sum_{i=1}^{N}U_i\|_H^2=\mathbf{E}\sum_{i=1}^{N}\|U_i\|_H^2$

## Proof of Theorem 2 (convergence to GFF)

Observe $\Psi \equiv Z$ is a stationary solution to the limit SPDE

$$\partial_t \Psi = (\Delta - m)\Psi - \lambda :\mathbf{E}(\Psi^2): \Psi + \xi \qquad :\mathbf{E}(\Psi^2): = \mathbf{E}\Psi^2 - \mathbf{E}Z^2$$

Let $\Phi_i = Z_i + Y_i$ be stationary solution (invariant law is the QFT)

Lemma. For $m$ large or $\lambda$ small, one has $\mathbf{E}\|Y_i\|_{H^1}^2 \lesssim 1/N$.

Proof: By symmetry,

$$\mathbf{E}\|Y_i\|_{H^1}^2 = \frac{1}{N}\sum_{j=1}^{N} \mathbf{E}\|\nabla Y_j\|_{L^2}^2 + \frac{1}{N}\sum_{j=1}^{N} \mathbf{E}\|Y_j\|_{L^2}^2$$

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$$\frac{1}{N}\sum_{j=1}^{N}\mathbf{E}\|\nabla Y_j\|_{L^2}^2 + \frac{m}{N}\sum_{j=1}^{N}\mathbf{E}\|Y_j\|_{L^2}^2 + \frac{\lambda}{N^2}\mathbf{E}\Big\|\sum_{i=1}^{N}Y_i^2\Big\|_{L^2}^2$$

$$\leq \lambda\mathbf{E}[\mathcal{R}] + \lambda\mathbf{E}\Big[\mathcal{D}\cdot\frac{1}{N}\sum_{j=1}^{N}\|Y_j\|_{L^2}^2\Big] \qquad \leq C$$

$$\mathcal{R} = \frac{1}{N^3}\sum_i \|\sum_j \nabla^{-\alpha} :Z_j^2 Z_i: \|_{L^2}^2 \qquad \sim 1$$

$$\mathbf{E}[\mathcal{R}] = \frac{1}{N^3}\sum_i\sum_{j_1}\sum_{j_2}\mathbf{E}\Big\langle \nabla^{-\alpha} :Z_{j_1}^2 Z_i: , \nabla^{-\alpha} :Z_{j_2}^2 Z_i: \Big\rangle \sim 1/N$$

$$\frac{1}{N}\sum_{j=1}^{N}\mathbf{E}\|\nabla Y_j\|_{L^2}^2+(m-\lambda\mathbf{E}[\mathcal{D}])\frac{1}{N}\sum_{j=1}^{N}\mathbf{E}\|Y_j\|_{L^2}^2+\frac{\lambda}{N^2}\mathbf{E}\Big\|\sum_{i=1}^{N}Y_i^2\Big\|_{L^2}^2$$

$$\leq \lambda\mathbf{E}[\mathcal{R}]+\lambda\mathbf{E}\Big[(\mathcal{D}-\mathbf{E}[\mathcal{D}])\cdot\frac{1}{N}\sum_{j=1}^{N}\|Y_j\|_{L^2}^2\Big]$$

$$\leq \frac{1}{N}+\frac{\lambda}{4N^2}\mathbf{E}\Big(\sum_{j=1}^{N}\|Y_j\|_{L^2}^2\Big)^2+C\mathbf{E}\Big[(\mathcal{D}-\mathbf{E}[\mathcal{D}])^2\Big]$$

e.g.

$$\mathcal{D}-\mathbf{E}[\mathcal{D}]=\frac{1}{N^2}\sum_{ij}\Big(\|:Z_iZ_j:\|_{C^{-\alpha}}^2-\mathbf{E}\|:Z_iZ_j:\|_{C^{-\alpha}}^2\Big)$$

**Rmk.** Theorem also holds in $d=3$ [S.-Zhu-Zhu, 2021], proof based on technique by Gubinelli-Hofmanova 2020 for $\Phi^4_{d=3}$.

## Proof of observable correlation formula

Theorem 3. Let $\mathcal{O} = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} :\Phi_i^2:$ , $G_N(x-y) = \mathbf{E}(\mathcal{O}(x)\mathcal{O}(y))$

$$\widehat{G_N} \overset{N\to\infty}{=} \frac{2\widehat{C^2}}{1+2\widehat{C^2}} \qquad C = (m-\Delta)^{-1}$$

Lemma (algebraic step).

$$(1 + \frac{2(N+2)}{N}\widehat{C^2})\widehat{G_N} = 2\widehat{C^2} + \widehat{Q_N}/N$$

where

$$Q_N(x-z) = -\int C(x-y)C(x-z)\mathbf{E}\Big( :\Phi_i \sum_j \Phi_j^2(y): \Phi_i(z)\Big)dy$$

$$+ 6 \text{ pt correlation}$$

Lemma (analysis step). $\widehat{Q_N}/N \to 0$ as $N \to \infty$.

Dyson-Schwinger: write $\mathbf{\Phi}^2 \stackrel{\text{def}}{=} \sum_{i=1}^{N} \Phi_i^2$

$$\int C(x-z)\mathbf{E}\Big[\Phi_1(x) :\Phi_1\mathbf{\Phi}^2(z): \ :\mathbf{\Phi}^2(y): \Big]dz$$
$$= -\mathbf{E}[ :\mathbf{\Phi}^2(x): \ :\mathbf{\Phi}^2(y): ] + 2NC(x-y)\mathbf{E}[\Phi_1(x)\Phi_1(y)]$$

$$\frac{1}{N}\int C(x-z_1)C(x-z_2)\mathbf{E}\Big[ :\Phi_1\mathbf{\Phi}^2(z_1): \ :\Phi_1\mathbf{\Phi}^2(z_2): \ :\mathbf{\Phi}^2(y): \Big]dz_1 dz_2$$
$$= -2\int C(x-y)C(x-z)\mathbf{E}\Big[ :\Phi_1\mathbf{\Phi}^2(z): \Phi_1(y)\Big]\, dz$$
$$+ \frac{N+2}{N}\int C(x-z)^2\mathbf{E}\Big[ :\mathbf{\Phi}^2(y): \ :\mathbf{\Phi}^2(z): \Big]dz$$
$$- \int C(x-z)\mathbf{E}\Big[\Phi_1(x) :\Phi_1\mathbf{\Phi}^2(z): \ :\mathbf{\Phi}^2(y): \Big]dz$$

$$Q_N(x-z) = -\int C(x-y)C(x-z)\mathbf{E}\Big( :\Phi_i \sum_j \Phi_j^2(y): \Phi_i(z)\Big) dy$$

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**Lemma.** $\widehat{Q_N}/N \to 0$ as $N \to \infty$.

**Proof:**

$$\mathbf{E}\Big( :Z_i \sum_j Z_j^2(y): Z_i(z)\Big) = 0$$

$$\mathbf{E}\Big(\sum_i \|Y_i\|_{L^2}^2\Big)^p < C \qquad \mathbf{E}\|Y_i\|_{L^p}^p < N^{\frac{p}{2}}$$

Proof of $\mathbf{E}\left(\sum_i \|Y_i\|_{L^2}^2\right)^p < C$:

Let $F = \frac{1}{N}\|\sum_{i=1}^N Y_i^2\|_{L^2}^2$ and $U = \sum_{i=1}^N \|Y_i^2\|_{L^2}^2$

$$\mathbf{E}[U^{q-1}F] + \mathbf{E}[U^q] \leq C + CN^{-1/2}(\mathbf{E}U^{q+1})^{q/(q+1)}$$

- Use term $\mathbf{E}[U^{q-1}F]$ and $F \geq N^{-1}U^2$ to get $\mathbf{E}[U^q] \leq N^{(q-1)/2}$
- Use term $\mathbf{E}[U^q]$ we get $\mathbf{E}[U^q] \leq N^{\frac{q^2}{2(q+1)} - \frac{1}{2}}$
- Iterate.

Future questions.

1. All $m \geq 0$, $\lambda > 0$?
2. Entire space, critical exponents etc.
3. Higher order $1/N$ expansions?
4. Other QFT models or SPDE systems?

Thank you for your attention!